Can quantum regression theorem be reconciled with quantum fluctuation dissipation theorem ?

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(February 1, 2008)

Abstract

In the attempt to derive the regression theorem from the fluctuation dissipation theorem several authors claim the violation of the former theorem in the quantum case. Here we pose the question: does it exists a quantum fluctuation dissipation theorem (QFDT) in its conventional interpretation ? It is shown that the relation usually called as the QFDT is the condition of detailed macroscopic energetic balance. Following this interpretation the existing conflict between the two theorems in the quantum case is removed.
PACS numbers: 03.65.Ca, 05.30.-d, 05.40.+j

Typeset using REVTEX

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I. INTRODUCTION

Under thermal equilibrium conditions the behavior of fluctuations of macroscopic observables of a physical system is governed by relationships which are formulated usually in terms of the regression theorem (the so called Onsager hypothesis [1]) and the fluctuation-dissipation theorem [2-6] (also known as the Nyquist relation). The former pertains to the time domain and states that the relaxation of a correlation of fluctuations is described by the same law governing the irreversible processes of the observable quantity itself. The latter pertains to the frequency domain and interrelates in some universal way the spectral characteristics of fluctuations and linear response (i.e. dissipation) of an observable of the physical system. Often, the fluctuation dissipation theorem is written as:

$$S_{xx}(\omega) = g^s(\omega) Im\{\alpha_x(\omega)\} \tag{1}$$

where $S_{xx}(\omega)$ is the spectral density of fluctuations of the observable x, $Im\{\alpha_x(\omega)\}$ the imaginary part of the generalized susceptibility which is responsible for the relaxation, and

$$g^s(\omega) = \hbar coth(\frac{\hbar\omega}{2kT}) = 2\hbar(\overline{n} + \frac{1}{2}) \tag{2}$$

is the Planck relating factor describing the average energy of the thermal radiation of the field mode with frequency ω.

In classical case $\hbar \to 0$, the two theorems give the same description of the spectrum of thermal fluctuations. By contrast, within the more general quantum case there appears a conflict between these two theorems since they predict different behavior of $S_{xx}(\omega)$. The quantum regression theorem (QRT) claims that the spectrum of fluctuations is determined only by the permitted transitions between energetic states of the system. In turn, the quantum fluctuation dissipation theorem (QFDT) states, that besides the eigenfrequencies of the system (as it required by the QRT) in the spectrum of the fluctuations there exist additionally the so called Matsubara frequencies, $\Omega_n = i\frac{2\pi kT}{\hbar}n$, where $n = \pm1, \pm2, ...$ The origin of these frequencies is related with the poles of the Planck factor $g(\omega)$.

This conflict is usually interpreted as a violation of the QRT (see, for example, Refs. [7-9]). In its most evident form such a violation is demonstrated in Ref. [9], where the conclusion statement announced that "there is no quantum regression theorem". The proof of the general character of such a statement is based on the fact that the violation of QRT follows from QFDT. However, a proof that QRT is valid independently of QFDT was given by Lax [10] on the basis of the general principles of quantum statistics (see also Refs. [11-13]). Since in Refs. [8,9] it is claimed that QFDT contradicts the validity of QRT, we argue that the origin of such a conflict is related with QFDT and its interpretation (see also Ref. [14]).

The aim of this article is to address this issue by considering the origin of such a conflict from a formal mathematical point of view.

II. FORMAL DERIVATION OF THE QFDT

In the operator representation, the symmetrized correlation function of the fluctuations of an observable x is written as [4-6]:

$$C_{xx}(\tau) = \frac{1}{2}Tr\{\hat{\rho}_s[\hat{x}(\tau)\hat{x}(0) + \hat{x}(0)\hat{x}(\tau)]\} \tag{3}$$

while the corresponding linear response function is given by Kubo formula [4-6]:

$$\alpha_x(\tau) = \frac{i}{\hbar}\Theta(\tau)Tr\{\hat{\rho}_s[\hat{x}(\tau)\hat{x}(0) - \hat{x}(0)\hat{x}(\tau)]\} \tag{4}$$

where $\hat{\rho}_s$ is the density operator which describes some stationary state of the physical system under test characterized by Hamiltonian $\hat{H}_s$, and $\Theta(\tau)$ is the unit step function.

The linear response described by Eq. (4) implies that the interaction between the system and the radiation is determined by the semi-classical Hamiltonian $\hat{V} = -\hat{x}f(t)$, where $f(t)$ is a classical force. By performing the Fourier transform of Eqs. (3) and (4), in accordance with ref. [6] one obtains:

$$\begin{bmatrix} S_{xx}(\omega) \\ \hbar Im\{\alpha_x(\omega)\} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} J_{xx}(\omega) + J_{xx}(-\omega) \\ J_{xx}(\omega) - J_{xx}(-\omega) \end{bmatrix} \tag{5}$$

where

$$J_{xx}(\omega) = \int_{-\infty}^{\infty} Tr\{\hat{\rho}_s \hat{x}(\tau)\hat{x}(0)\} e^{i\omega\tau} = 2\pi \sum_{m,n} \rho_n |x_{mn}|^2 \delta(\omega_{mn} - \omega) \qquad (6)$$

is the spectral density corresponding to the one-directional in time correlation function, $Tr\{\hat{\rho}_s \hat{x}(\tau)\hat{x}(0)\}$ and $J_{xx}(-\omega)$ corresponds to $Tr\{\hat{\rho}_s \hat{x}(0)\hat{x}(\tau)\}$, ρ_n is the probability to find the system in the eigenstate with energy E_n, x_{mn} is the matrix representation of the operator $\hat{x}$ and $\omega_{mn} = (E_m - E_n)/\hbar$ is the frequency associated with the transition between the energetic states E_m and E_n.

Under thermal equilibrium $\hat{\rho}_s = exp(-\beta\hat{H}_s)/Tr\{exp(-\beta\hat{H}_s)\}$ with $\beta = 1/(kT)$. For the derivation of the QFDT an explicit expression for $J_{xx}(\omega)$ is not necessary, it is sufficient the existence of the quantum spectral relation [6]:

$$J_{xx}(-\omega) = e^{-\beta\hbar\omega} J_{xx}(\omega) \qquad (7)$$

By substituting Eq. (7) into Eq. (5) one obtains:

$$\begin{bmatrix} S_{xx}(\omega) \\ \hbar Im\{\alpha_x(\omega)\} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 + p(\omega) \\ 1 - p(\omega) \end{bmatrix} J_{xx}(\omega) \qquad (8)$$

where $p(\omega) = exp(-\beta\hbar\omega)$. According to Ref. [3-6], the QFDT is then obtained by excluding the factor $J_{xx}(\omega)$ which is common to both $S_{xx}(\omega)$ and $Im\{\alpha_x(\omega)\}$. As a consequence of such a derivation, the zeros of $Im\{\alpha_x(\omega)\}$ determined by the factor $1 - p(\omega) = 0$ (see Eq. (8)) become the poles of the Planck factor in Eq. (1), i.e., they originate the Matsubara frequencies and, in turn, the QRT-QFDT conflict. Thus, the poles of the Planck factor at the Matsubara frequencies can not be considered as independent of the frequency dependence of $Im\{\alpha_x(\omega)\}$ which, in accordance with Eq. (8), has zeros at the same frequencies:

$$Im\{\alpha_x(\omega)\}\Big|_{\omega=\Omega_n} = 0 \qquad (9)$$

Therefore, from a mathematical point of view, in this case the right-hand side of Eq. (1) contains an indefinite form of $\frac{0}{0}$-type.

Now we pose the following open question: does the constraint given by Eq. (9) merely represent the formal requirement necessary for a rigorous derivation of the QFDT, or bring a proper physical meaning ?

An attempt to answer this question is detailed in the following section.

III. INTERRELATION BETWEEN THE QFDT AND THE PRINCIPLES OF ENERGY BALANCE

As physical model we shall consider a sufficiently large isolated system subdivided into two subsystems. The first corresponds to some physical system under test, the second represents the surrounding world. In this case the total Hamiltonian can be written as: $\hat{H} = \hat{H}_S + \hat{H}_T + \hat{V}$, where $\hat{H}_S$ and $\hat{H}_T$ are Hamiltonians of subsystems, and $\hat{V}$ describes the interaction between these subsystems. By using the standard procedure [15] to construct the master equations for the statistical operators of each subsystem, $\hat{\rho}_i$ ($i = S, T$), and assuming that the interaction is weak one obtains the following equation for time variations of the average energy $< \hat{H}_i >$ in the i-th subsystem:

$$\frac{d}{dt} < \hat{H}_i >= \frac{1}{\hbar^2} \int_0^\infty Tr_{S+T}\{\hat{\rho}_S\hat{\rho}_T[[\hat{H}_i, \hat{V}], \hat{V}(-\tau)]\}d\tau \tag{10}$$

where $< \hat{H}_i >= Tr_i\{\hat{\rho}_i\hat{H}_i\}$, $\hat{V}(-\tau) = exp[-\frac{i}{\hbar}(\hat{H}_S + \hat{H}_T)\tau]\hat{V}exp[\frac{i}{\hbar}(\hat{H}_S + \hat{H}_T)\tau]$.

The energy exchange described by Eq. (10) satisfies the conservation law for the total energy of an isolated system in the form corresponding to the assumption of a weak interaction: $\frac{d}{dt} < \hat{H}_S + \hat{H}_T >= 0$. By assuming that the interaction Hamiltonian $\hat{V} = -\hat{x}\hat{f}$ is linear and factorized with respect to the variables of both subsystems the matrix representation in the right-hand side of Eq. (10) takes the form:

$$\frac{d}{dt}\begin{bmatrix} < \hat{H}_S > \\ < \hat{H}_T > \end{bmatrix} = \frac{\pi}{\hbar}\sum_{m,n}^S\sum_{M,N}^T |x_{mn}|^2|f_{MN}|^2\begin{bmatrix} \omega_{mn}^S \\ \omega_{MN}^T \end{bmatrix}(\rho_m^S\rho_M^T - \rho_n^S\rho_N^T)\delta(\omega_{mn}^S + \omega_{MN}^T) \tag{11}$$

From Eq. (11) one directly obtains the condition of the microscopic detailed energetic balance (MiDEB):

$$\rho_m^S \rho_M^T = \rho_n^S \rho_N^T \qquad or \qquad \frac{\rho_m^S}{\rho_n^S} = \frac{\rho_N^T}{\rho_M^T} \tag{12}$$

which must be satisfied with respect to only those energy states of both subsystems that are directly involved in the interaction, i.e., when $\omega_{mn}^S = \omega_{MN}^T$, $|x_{mn}|^2 \neq 0$, $|f_{MN}|^2 \neq 0$.

Now, let us formulate the conditions of the energy balance at the macroscopic level of description. For this sake in Eq. (11) we replace the term $\delta(\omega_{mn}^S + \omega_{MN}^T)$ by $\int \delta(\omega_{mn}^S - \omega)\delta(\omega_{MN}^T + \omega)d\omega$ and rewrite Eq. (11) by using the matrix representation of the asymmetric spectral density $J(\omega)$ given by Eq. (6):

$$\frac{d}{dt} < \hat{H}_S >= -\frac{d}{dt} < \hat{H}_T >= \frac{1}{4\pi\hbar} \int \omega [J_{xx}(-\omega)J_{ff}(\omega) - J_{xx}(\omega)J_{ff}(-\omega)]d\omega \tag{13}$$

From Eq. (13) we obtain the condition of macroscopic detailed energy balance (MaDEB) as:

$$J_{xx}(-\omega)J_{ff}(\omega) = J_{xx}(\omega)J_{ff}(-\omega) \tag{14}$$

which requires to be fulfilled for any value of the current frequency ω. We notice that the condition given by Eq. (14) is not the only form which can be used to express such a detailed balance. For example, by using the definitions of $Im\{\alpha_x(\omega)\}$ and $S_{xx}(\omega)$ given by Eq. (5) it is easy to show that Eq. (14) can be rewritten in an equivalent form as:

$$Y_f(\omega)Im\{\alpha_x(\omega)\} = Y_x(\omega)Im\{\alpha_f(\omega)\} \tag{15}$$

where for $Y_i(\omega)$ one can use any of the three spectral densities, namely: $J_{ii}(\omega)$, $J_{ii}(-\omega)$ and $S_{ii}(\omega)$ $(i = x, f)$. The equivalence of Eqs. (14) and (15) from the point of view of the formal determination of $J(\omega)$, $S(\omega)$ and $Im\{\alpha(\omega)\}$ given by Eqs. (5) and (6) does not mean their entire macroscopic equivalence. This is related to the fact, that with respect to Eq. (14), Eq. (15) involves additionally the notion of the macroscopic function of linear response, $\alpha_i(\omega)$, all properties of which cannot be obtained from Eqs. (5) and (6) only.

Let us consider the conditions when the MiDEB given by Eq. (12) and the MaDEB given by Eqs. (14) and (15) can be considered as equivalent descriptions of the energy balance

under steady state. In the following we shall use the formulation of balance conditions through some ratio of characteristics of a subsystem [see the right-hand side version of Eq. (12)] since such ratios sometimes are universal functions which are independent from the internal properties of the interacting subsystems. Let us rewrite Eq. (14) in the form:

$$\frac{J_{xx}(-\omega)}{J_{xx}(\omega)} = \frac{J_{ff}(-\omega)}{J_{ff}(\omega)} \equiv p(\omega) \quad or \quad J_{ii}(-\omega) = p(\omega) J_{ii}(\omega) \tag{16}$$

where $p(\omega)$ is common for both subsystems factor which is some single-valued function of the current frequency and it satisfies the condition $p(-\omega) = p^{-1}(\omega)$. It is easy to show that the equivalence of the energy balance description given by Eq. (12) and (16) is satisfied if $p(\omega)$ can be defined from the microscopic level as:

$$\frac{\rho_m^S}{\rho_n^S} = \frac{\rho_N^T}{\rho_M^T} = p(\omega)\Big|_{\omega=\omega_{mn}=\omega_{NM}} \tag{17}$$

Let us rewrite the MaDEB condition given by Eq. (15) in a form analogous to Eq. (16)

$$\frac{Y_x(\omega)}{Im\{\alpha_x(\omega)\}} = \frac{Y_f(\omega)}{Im\{\alpha_f(\omega)\}} \equiv g^Y(\omega) \quad or \quad Y_i(\omega) = g^Y(\omega) Im\{\alpha_i(\omega)\} \tag{18}$$

Here the function of current frequency $g^Y(\omega)$ will depend on which spectral density [i.e., the symmetric $S(\omega)$ or asymmetric $J(\pm\omega)$] is used to formulate the balance conditions. The fulfillment of Eq. (16) allows us to represent the frequency dependence of $S_{ii}(\omega)$ and $Im\{\alpha_i(\omega)\}$ of both subsystems in a form entirely analogous to thermal equilibrium [see Eq. (8)] with the only difference that now $p(\omega)$ is not necessarily given by the thermal value. From the above it is easy to see that all the functions $g^Y(\omega)$ in Eq. (18) are determined by by the frequency dependence of $p(\omega)$ only:

$$g^Y(\omega) = \frac{\hbar}{1 - p(\omega)} \begin{cases} 1 & , \quad for \quad J(\omega) \\ p(\omega) & , \quad for \quad J(-\omega) \\ 1 + p(\omega) & , \quad for \quad S(\omega) \end{cases} \tag{19}$$

When $p(\omega)$ is a universal function of frequency [e.g., in thermal equilibrium when $p(\omega) = exp(-\beta\hbar\omega)$] there is the possibility to formulate the MaDEB conditions in terms of expressions which relate in some universal way the macro-characteristics of only one of the

interacting subsystem [see the right-hand side expressions in Eqs. (16) and (18)]. However, in so doing, it is easy to loose the physical meaning of these expressions.

In thermal equilibrium, Eq. (18) allows the QFDT to be given an alternative physical interpretation with respect to the conventional one. Indeed, by replacing in Eq. (18) $Y_i(\omega)$ with the symmetrical spectral density $S_{ii}(\omega)$ one obtains the MaDEB condition [right-hand side expression in Eq. (18)] in a form which is identical for both the subsystems and which coincides with the conventional form of the QFDT given by Eq. (1). This allows us to conclude that the QFDT describes the detailed energetic balance between the interacting physical systems under thermal equilibrium. As a consequence, the usual interpretation that the frequency dependences of the Planck factor $g^s(\omega)$ [see Eq. (2)] and of the imaginary part of the generalized susceptibility $Im\{\alpha_i(\omega)\}$ are independent (the source of the QRT-QFDT conflict) is in contradiction with both the MiDEB and MaDEB principles. When these principles are fulfilled, the poles of $g^s(\omega)$ and the zeros of $Im\{\alpha_i(\omega)\}$ are determined by the same factor $1 - p(\omega)$ [see Eqs. (19) and (8), respectively]. The neglect of this property when treating fluctuation phenomena corresponds in essence to the violation of the MiDEB and MaDEB principles and, hence, it implies the violation of the condition of stationarity at least at the Matsubara frequencies $\omega = \Omega_n$ where $p(\Omega_n) = 1$.

The formulation of the balance conditions represented in terms of a ratio of macro-characteristics of one or of both subsystems can serve as a source of incorrect interpretation of the MaDEB principle and, as a consequence, can lead to the violation of the energy conservation law for subsystems with partially overlapping energetic spectra. The typical example is the interaction between the harmonic oscillator with eigenfrequency ω_s and the thermal bath characterized by a quasi-continuous spectrum. By using for the matrix elements of oscillator the relations [15]: $|x_{mn}|^2 = (1/\omega_s)\gamma_s^2 \ \hbar(n+1)\delta_{m,n+1}$ and $\omega_{mn}^S = \omega_s(m-n)$, from Eq. (6) one obtains asymmetric spectral densities $J(\pm\omega)$ represented as:

$$\begin{bmatrix} J(-\omega) \\ J(\omega) \end{bmatrix} = 2\pi \frac{\gamma^2(\omega) \ \hbar}{\omega} \begin{bmatrix} \overline{N}(\omega) \\ \overline{N}(\omega) + 1 \end{bmatrix} \delta(\omega_s - \omega) \tag{20}$$

where $\overline{N}(\omega_s) = \sum_{n=0}^{\infty} n\rho_n = [exp(\beta\hbar\omega_s) - 1]^{-1}$, $\gamma^2(\omega_s)$ is the electro-dipole matrix element

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square which can depend upon the oscillator frequency. In Eq.(20), due to the properties of the δ-function we have performed the replacement usually used in such situations when dependence of $\gamma^2(\omega_s)$ and $\overline{N}(\omega_s)$ on ω_s is replaced by their dependence on the current frequency ω. In this way, from Eq. (20) one obtains:

$$\frac{J(-\omega)}{J(\omega)} = p(\omega) = \frac{\overline{N}(\omega)}{\overline{N}(\omega) + 1} \tag{21}$$

which defines in explicit form $p(\omega)$ in the whole frequency range, while the spectrum of a single oscillator is defined at the frequency $\omega = \omega_s$ only, i.e. it is described by a δ-function which in Eq. (21) disappeared from explicit consideration.

By considering the thermal bath, also in this case it is used the harmonic oscillator model which describes one mode of the radiation field. To obtain the spectral densities $J(\pm\omega)$ for the thermal bath it is sufficient to perform in the right-hand side of Eq. (20) a summation over all the possible modes, that is over subindex s. In so doing, the dependence of $\gamma^2(\omega_s)$ and $\overline{N}(\omega_s)$ on ω_s but not on ω can remain. By using the standard procedure for systems with a quasi-continuous spectrum, i.e., by replacing $\sum_s$ by $\int G(\omega_s)d\omega_s$, where $G(\omega_s)$ is the density of the radiation mode with frequency ω_s in the interval $d\omega_s$, one obtains:

$$\begin{bmatrix} J(-\omega) \\ J(\omega) \end{bmatrix} = 2\pi \frac{G(\omega)\gamma^2(\omega)\hbar}{\omega} \begin{bmatrix} \overline{N}(\omega) \\ \overline{N}(\omega) + 1 \end{bmatrix} \tag{22}$$

From Eq. (22) one can again obtain Eq. (21), which is now valid in the whole frequency range. When a single oscillator interacts with the thermal bath, the interaction takes place not in the whole frequency range, as it would follow from Eqs. (21) or (18), but only at the oscillator eigen-frequency, where the energy exchange is only possible.

IV. CONCLUSIONS AND OPEN QUESTIONS

Here we have shown that the QFDT expressed in its usual form of Eq. (1) in essence represents the macroscopic principle of detailed energy balance between a physical system and the surrounding world interacting with it. Such an interpretation, and the related restrictions

concerning the frequency dependence of the imaginary part of the generalized susceptibility of the system [see Eq. (9)], leads to two main consequences. First, the announced conflict between QRT and QFDT related with the Matsubara frequencies is reconciled. Second, some conventional notions as macroscopic dissipation and relaxation closely related to the QFDT should be revisited. Indeed, these notions cannot be treated as internal characteristics of one of the two interacting subsystems which compose the total isolated system.

In the framework of the macroscopic approach where the energy dissipation is described by $Im\{\alpha(\omega)\}$, the energy conservation law under the energy balance implies that the power dissipated by one of subsystems from another must be returned back, i.e. it is equal to the power dissipated by another subsystem from the first. The form of the MaDEB given by Eq. (15) just express this mathematically, by claiming that the spectra of the power dissipated by each subsystem from another are equal. Therefore, with respect to one subsystem, one side of Eq. (15) can be treated as the dissipated power and another side as the power returned back, i.e. as emitted power. This is illustrated by the following example, which considers the system interaction with thermal bath. By using Eq. (22), the MaDEB conditions given by Eq. (15) take the form:

$$2 \hbar\omega \left[\frac{\overline{N}(\omega) + \frac{1}{2} \pm \frac{1}{2}}{\overline{N}(\omega) + \frac{1}{2}} \right] Im\{\alpha_x(\omega)\} = \omega \left[\begin{array}{c} J_{xx}(\pm\omega) \\ S_{xx}(\omega) \end{array} \right] \tag{23}$$

The left-hand side of Eq. (23) describes the power dissipated by the system when this power is provided by the thermal bath. The right-hand side of Eq. (23) describes the power which the system returns back to the thermal bath. If the only photon part $\hbar\omega\overline{N}(\omega)$, of the full energy of the field is involved into the absorption process, then the returning spectrum, $J_{xx}(-\omega)$, corresponds to the spectrum of the spontaneous emission processes occurring in the system. This is easy shown by using the explicit form of $J(-\omega)$ given by Eq. (6). If now one reverses time, i.e. replace ω by $-\omega$, the energy of the thermal bath which is involved into dissipation process will be $\hbar\omega[\overline{N}(\omega) + 1]$. This means that with respect to one of the interacting subsystems the processes of energy exchange in equilibrium conditions shows an asymmetry under time reversal. If the system has absorbed the full energy of the field

together with the zero-field value, $\hbar\omega[\overline{N}(\omega) + \frac{1}{2}]$ (the case of the QFDT), to fulfill MaDEB it would be necessary that the system returns back the same full energy. In this case, the emitted spectrum which returns back would be described by the symmetric spectral density $S_{xx}(\omega)$. Here the following question arises, which spectrum of fluctuations and correspondingly which correlation function are experimentally measured: the asymmetric or symmetric ones ?

In applications, the QFDT is often used to describe the relaxation phenomena in systems interacting with a thermal bath. In this interpretation as one of the formulation of the MaDEB conditions there appears a question about the correctness of such a use. Indeed, if the relaxation process implies that the system is approaching the stationary state, it means that the system is not in a stationary state. It is evident that under nonstationary conditions all the relations considered above loose their meaning, and any attempt to use them will imply a violation of the energy conservation law. Thus, the QFDT can be applied for the description of only those relaxation phenomena which occur under energy equilibrium conditions, as described by the QFDT itself.

The MaDEB conditions lead to another important property of the relaxation processes: the description of the relaxation in the interacting subsystems which compose an isolated system cannot be considered in independent way. From the definition of the linear response function $\alpha_i(\omega)$ it follows that the spectral densities of the fluctuations in each of the subsystems must be interrelated as $S_{ii}(\omega) = |\alpha_i(\omega)|^2 S_{jj}(\omega)$ where $i \neq j$. Together with the MaDEB in the form given by Eq. (18) this leads to additional restrictions put on the frequency dependence of $Im\{\alpha_i(\omega)\}$ pertaining to each of the two subsystems:

$$Im\{\alpha_i^{-1}(\omega)\} = Im\{\alpha_j(\omega)\} \qquad (24)$$

where $Im\{\alpha_i^{-1}(\omega)\} = Im\{\alpha_i(\omega)\}/|Im\{\alpha_i(\omega)\}|^2$ is the relaxation law in one of the subsystems, which in accordance with Eq. (24) is determined by the imaginary part of the generalized susceptibility of the other subsystem. For example, in the presence of an interaction of some systems with a thermal bath characterized by the radiation mode density

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$G(\omega) = \omega^2 \pi^{-2} c^{-3}$ [15] the relaxation in the system must be described by the law:

$$Im\{\alpha_i^{-1}(\omega)\} = \frac{1}{2\hbar}[J_{ff}(\omega) - J_{ff}(-\omega)] = \frac{\gamma^2(\omega)}{\pi c^3}\omega \qquad (25)$$

If the electrodipole matrix element square, γ^2, is independent of ω, one obtains the usual law of viscous friction.

ACKNOWLEDGMENTS

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